approved 11-7-96

Technical Report 1729
October 1996

Quotient Quadrature Amplitude Modulation (QQAM) for Fading Channels

M. J. Shensa

CONTENTS

FIGURES

1 INTRODUCTION

Differential coding presents a standard technique for combating channel instability in wireless communications [1]–[7]. Moreover, differentially encoded signals do not require synchronization for suppressed carriers, thereby avoiding additional noise from devices such as phase-locked loops. This approach is particularly effective when the basic modulation is M-ary phase-shift keying (MPSK). in which case it is labeled DPSK (differential MPSK). On the other hand, if the channel is relatively benign and fading is not an issue, one may obtain higher bit rates (as well as an improved signal-to-noise ratio (SNR) performance over MPSK) for an equivalent bandwidth by using M-ary quadrature amplitude modulation (MQAM). A variation, particularly used to combat multiplicative Rayleigh fading while still retaining MQAM, is differential MQAM (i.e., DQAM). In the case of DQAM, the differential encoding removes, or at least mitigates, the effects of channel fading upon the phase component of the MQAM signal. However, the amplitude is still vulnerable to the channel, and one must resort to approximate techniques that estimate the magnitude of fading (e.g., automatic gain control (AGC)) in order to extract the symbol amplitude [3].[1] Such methods fail if the fade rate becomes too large. That is, they suffer from a trade-off between the quality of the channel estimation and the speed at which it is tracked [7]. In this paper, we propose a technique called *quotient coding*, which is designed to remove channel effects from the symbol amplitude as well as its phase. In particular, we shall apply it to MQAM, resulting in a modulation that we refer to as Quotient Quadrature Amplitude Modulation (QQAM). Unlike DQAM, QQAM is just as effective at suppressing the effects of channel fading with respect to the entire symbol as DPSK is for the phase alone. Furthermore, it can be applied to arbitrary MQAM configurations.

Our discussion takes place at baseband. In fact, since pulse shape is irrelevant to the discussion, the description shall be entirely in terms of complex symbols. To develop our notation and set the context, we begin with a brief outline of MPSK, MQAM, and their differential counterparts. MPSK modulation may be summarized by [5]:

$$s \;=\; A\, e^{2\pi j(m-1)/M} \;=\; A\, e^{j\phi_m}, \tag{1a}$$

where A is real,

$$\phi_m \;\triangleq\; 2\pi j(m-1)\,/\,M; \qquad m = 1,\, \ldots,\, M\;, \tag{1b}$$

and M is the total number of symbols to be transmitted.

We shall assume that the channel in question can be modeled by multiplicative fading (typically Rayleigh [4]) and additive noise. In other words, the received signal takes the form $\alpha(t)s(t)+n(t)$, where $\alpha(t)$ is a complex number with appropriate time correlation, and $n(t)$ is noise. Thus, for example, to demodulate MPSK either one must estimate the phase of α, which is difficult when the fading is rapid, or one must modify the modulation scheme.

[1] Star 16QAM differentially encodes 1 bit of amplitude, but the technique does not generalize to 2 or more amplitude bits [1]–[3], [6], [7].

A standard technique is to use DPSK (differential MPSK), which may be implemented as follows: Let $d(t)$ be the transmitted DPSK signal, and define its amplitude $A_d(t)$ and phase $\theta(t)$ by

$$d(t) = A_d(t)\, e^{j\theta(t)}. \tag{2}$$

Note that $A_d(t)$ is real. Assume that $d(0)$ is known, for example that it equals 1. Then, define $d(t)$ as a function of its previous value, $d(t-1)$, and of the current symbol, $s(t)$, by

$$d(t) = A_d(t)\, e^{j(\phi(t)+\theta(t-1))} = \frac{s(t)}{A}\, d(t-1). \tag{3}$$

Let d_r be the received signal, and a "bar" indicate complex conjugation. The phase of the symbol $s(t)$ is recovered by

$$phase(\ s(t)\ ) = phase(\ d_r(t)\ \overline{d_r(t-1)}\ ), \tag{4}$$

and is essentially independent of the fading channel. That is, it is independent to the extent that the phase shift due to the channel, $phase(\ \alpha(t)\ \overline{\alpha(t-1)}\ )$, is approximately zero.

MQAM is a combination of amplitude and phase modulation. That is,

$$s = A_n\, e^{2\pi j(p-1)/P} = A_n\, e^{j\phi_p} \qquad n = 1, \ldots, N; \quad p = 1, \ldots, P, \tag{5}$$

where the total number of symbols, M, is given by $M = NP$. DQAM is implemented by differentially encoding the phase:

$$d(t) = s(t)\, \frac{d(t-1)}{||d(t-1)||}, \tag{6a}$$

and recovering the symbol by

$$s_r(t) = d_r(t)\, \frac{\overline{d_r(t-1)}}{||d_r(t-1)||}. \tag{6b}$$

But only the phase of $s_r(t)$ is impervious to the channel. The received amplitude, $|s_r(t)| = |\alpha(t)s(t)|$ requires channel estimation in order to extract the amplitude of s(t). This may be done by using averaging techniques such as AGC, but there is a performance trade-off inasmuch as one must average (a) enough to smooth out the dependence on A_n, but (b) over a short enough time period to track the channel.

2 QUOTIENT CODING

The basic modulation paradigm for quotient coding is quite simple. One modulates via the relationship

$$\tilde{q}(t) \;=\; s(t)\,\tilde{q}(t-1), \tag{7a}$$

and demodulates by

$$s_r(t) \;=\; \tilde{q}_r(t)\;/\;\tilde{q}_r(t-1). \tag{7b}$$

(The tildes appear in equations (7) to distinguish $\tilde{q}$ from our final result in equations (10) and (13) below.) It is clear that the multiplicative channel cancels out both with respect to phase and amplitude. That is, in the absence of noise,

$$\tilde{q}_r(t)\;/\;\tilde{q}_r(t-1) \;=\; (\alpha(t)\tilde{q}(t))\;/\;(\alpha(t-1)\tilde{q}(t-1)) \;\approx\; \tilde{q}(t)\;/\;\tilde{q}(t-1). \tag{8}$$

There is, however, a practical difficulty associated with equation (7a). If $|s(t)| \neq 1$, the transmitted power will experience large excursions caused by the variation in the amplitude of the transmitted symbol. In particular, $|\tilde{q}(t)| = |\prod_{t_i<t} s(t_i))|\;|\tilde{q}(0)|$. On the average, these excursions will average out. But there are still likely to be long sequences producing very small or very large transmitted powers. The former is sensitive to noise, the latter physically unrealizable. (The 2DASK differential amplitude scheme, which recognizes a "1" by a change in amplitude and a "0" otherwise, avoids this problem if it is implemented so that the transmitted symbol takes on exactly two amplitudes; but *this is only possible for* $N = 2$. When used in conjunction with 8PSK, such a scheme is generally referred to as Star DQAM [1], [2], [6], [7].) This dynamic range problem is alleviated by a simple, but critical, modification which we proceed to describe.

Let us return, momentarily, to the MQAM modulation of equation (5). Without loss of generality, we order the symbols by size,

$$A_1 \;<\; A_2 \ldots \;<\; A_N, \tag{9a}$$

and normalize the amplitudes so that

$$A_1 \;=\; 1. \tag{9b}$$

Let P_{max} be the maximum transmission power at baseband. The modified quotient coder is defined by

$$q(t) \;=\; \begin{cases} q(t-1)\;s(t) & |q(t-1)\;s(t)| \;\leq\; \sqrt{P_{max}}; \\ \\ q(t-1)\;/\;\overline{s(t)} & other. \end{cases} \tag{10}$$

A reasonable initialization value is $q(-1) = \sqrt{P_{max}}/2.0$. Now, let $P_r(t)$ be the instantaneous received power

$$P_r(t) \;\triangleq\; |q_r(t)|^2. \tag{11}$$

Note that in the absence of noise, $P_r(t) \,/\, P_r(t-1)$ is independent of the fading $\alpha(t)$, and that it equals $|s(t)|^2$ or $1 \,/\, |s(t)|^2$ according to equation (10). Since $A_1 = 1 > 1/A_2$, these two ratios correspond to two populations: that for which $\sqrt{P_r(t) \,/\, P_r(t-1)}$ is greater than or equal to A_1 and that for which it is less than or equal to $1/A_2$. (For the case $|s| = A_1$, the ratios of the powers are equal; i.e., $|s(t)|^2 = 1/|s(t)|^2 = 1$.)

Define

$$\begin{aligned} \eta &\triangleq 0.5\,(A_1 + \frac{1}{A_2}) \\ &= 0.5\,(1 + \frac{1}{A_2}). \end{aligned} \tag{12}$$

The demodulation scheme becomes

$$s_r(t) = \begin{cases} q_r(t) \,/\, q_r(t-1) & for\ P_r(t) \geq \eta^2\, P_r(t-1) \\ \overline{q_r(t-1)} \,/\, \overline{q_r(t)} & for\ P_r(t) < \eta^2\, P_r(t-1) \end{cases}, \tag{13}$$

which may also be written

$$s_r(t) = \begin{cases} q_r(t)\, \overline{q_r(t-1)} \,/\, P_r(t-1) & for\ P_r(t) \geq \eta^2\, P_r(t-1) \\ q_r(t)\, \overline{q_r(t-1)} \,/\, P_r(t) & for\ P_r(t) < \eta^2\, P_r(t-1) \end{cases}. \tag{14}$$

It should be remarked that the actual constellation transmitted, equation (10), is a non-linearly distorted version of the associated MQAM constellation equation (5). However, it does closely resemble MQAM locally, over the transmission of a few symbols (cf. Appendix). In fact, the detector behavior is roughly equivalent to an MQAM constellation under a mixture of of SNRs. In addition, one must take into account the probability of an error in the binary decision based on the threshold, η (cf., equation (13)).

3 AN EXAMPLE

To illustrate, we present an example with two amplitude bits and three phase bits; in other words, 32QQAM with $N = 4$ and $P = 8$. The constellation on which this example is based is pictured in figure 1. Its rings are equally spaced with amplitudes, A_i, given by the formula $A_i = 1 + 0.4\,(i - 1)$, and are individually Gray coded. The factor 0.4 was obtained by running several simulations and picking the best value. However, we have not made any serious attempt to optimize the constellation. There is no reason to assume that equally spaced rings or that annuli are the best geometry. For that matter, what is "best" depends on the application.

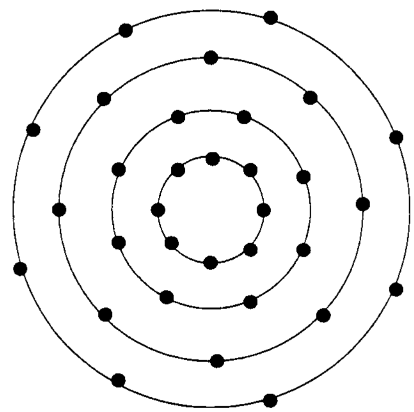


Figure 1. Underlying QQAM (and also the DQAM) constellation used in the simulation example. There are 3 bits assigned to phase and 2 bits to amplitude.

The DQAM modulation used for comparison was also based on the constellation of figure 1. To be fair, several runs were made to determine whether the ring locations should be adjusted, but it was found that the DQAM behavior was relatively insensitive to the ring spacing and that 0.4 was close to optimal under equal spacing. An AGC with an exponential window was used to handle fading. More precisely, the channel was tracked via $P_{av}(t_i) = (1 - \lambda)\, P_{av}(t_{i-1}) + \lambda\, |d_r(t)|^2$, where P_{av} represents the average power estimate. A value of $\lambda = 1/40 = 0.025$ was chosen. It corresponds to a window of 40 symbols. This value was originally determined by optimizing performance for the fade rate under study (0.0018 normalized Doppler[2]), but was found to be close to optimal even for very slow fading (fade rates of 10^{-5} and, perhaps, lower). Roughly speaking, it is about as short a window as one can use while still averaging sufficiently over the four possible amplitudes.

Figure 2 compares 32QQAM with 32DQAM in a fading environment with no additive noise ($E_b/N_0 = \infty$). The advantage of QQAM is rather remarkable. Except at extremely low fade rates (cf., figure 4), the AGC-dependent DQAM suffers much more from the fading than QQAM does. This huge performance gap is somewhat more subdued in the

[2]The normalized maximum Doppler frequency is defined as max_doppler_frequency/symbol_rate = f_d/f_{symbol}.

presence of noise. Figure 3 contains simulation results as a function of the signal-to-noise per bit E_b/N_0 for a fading channel where the maximum normalized Doppler frequency is fixed at 0.0018. At high SNR, where fading is the dominant effect, QQAM does much better than DQAM. More precisely, there is a crossover point at about 12 dB. Past this point, 32DQAM saturates, and 32QQAM gains dramatically in performance.

The other extreme from figure 2, no fading at all, is illustrated in figure 4. For completeness, we have also included performance curves for 32PSK, 32DPSK, and 32QAM. Since there is no fading, this case is effectively to the left of the crossover point, and 32DQAM significantly outperforms 32QQAM. The gap is rather large, but this is not surprising inasmuch as the 32DQAM of figure 4 was implemented by assuming perfect knowledge of the channel amplitude.

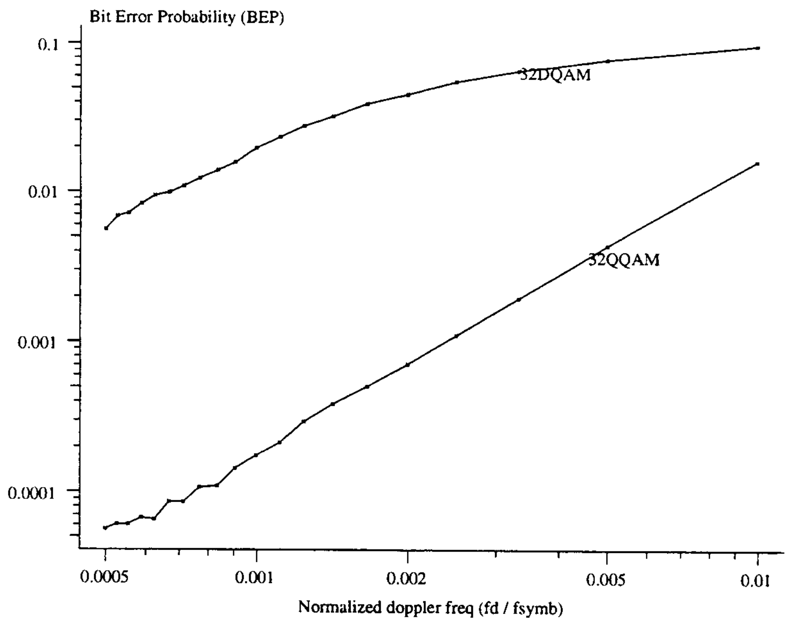


Figure 2. Comparison of 32QQAM and 32DQAM with 3 bits of phase and 2 bits of amplitude. The channel model consists of pure multiplicative Rayleigh fading [4] (i.e., no additive noise), and the x-axis is expressed as the normalized maximum Doppler frequency (i.e., f_d/f_symbol).

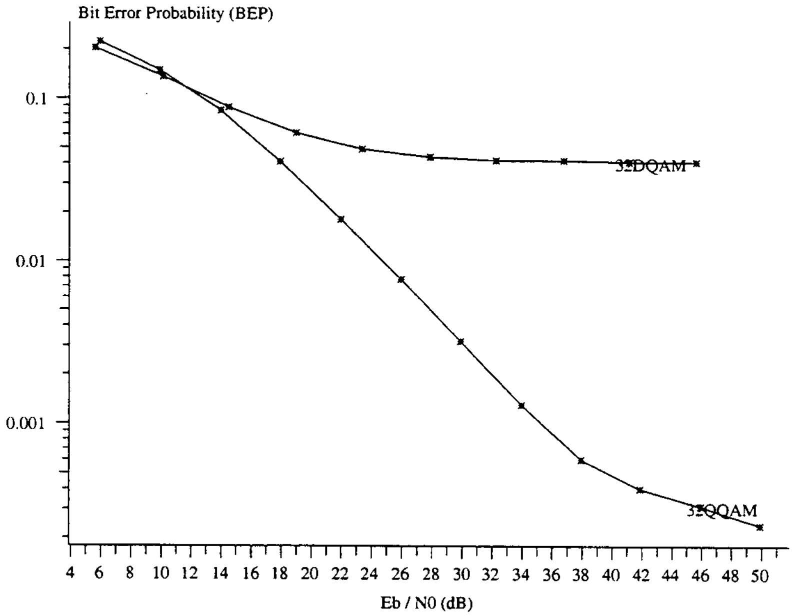


Figure 3. Comparison of 32QQAM and 32DQAM as a function of E_b/N_0. The channel model consists of multiplicative Rayleigh fading with a normalized Doppler frequency of 0.0018 plus additivie white gaussian noise (AWGN).

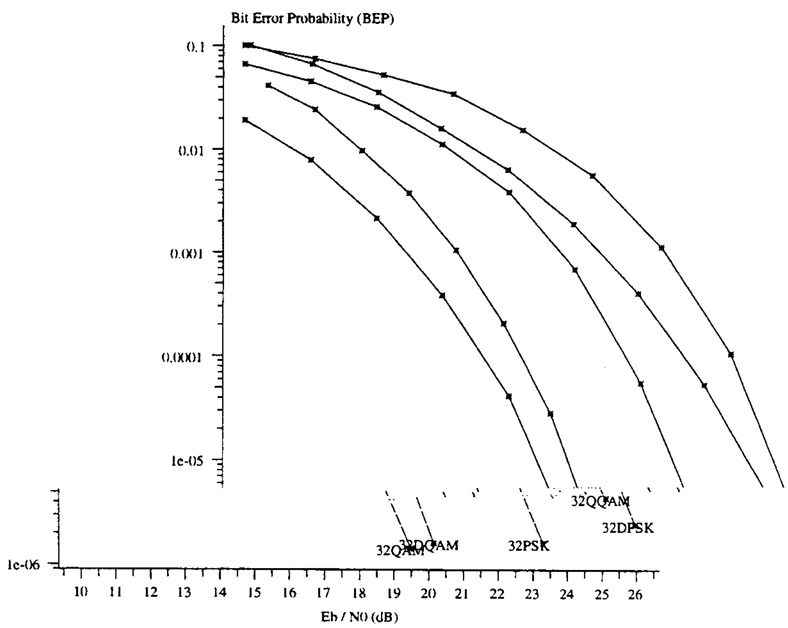


Figure 4. Performance of 32QQAM and several other modulation schemes in the presence of pure AWGN. No AGC was used for the 32DQAM (i.e., it was frozen at the true power). From left to right, the curves are 32QAM, 32DQAM, 32PSK, 32QQAM, and 32DPSK.

4 CONCLUSION

We have described a very general technique for removing the deleterious effects of multiplicative fading from most modulation schemes. Termed *quotient coding*, it determines the value to be transmitted by forming the quotient (or the product, when dictated by the current state) of the previous value with the current symbol. Quotient coded MPSK is equivalent to DPSK. The application of quotient coding to an annular 16QAM constellation produces a variant of Star DQAM [6]. Quotient coding of an arbitrary MQAM constellation results in a modulation form that we have named QQAM. The actual constellation transmitted is a nonlinear, distorted version of the associated MQAM constellation. Nevertheless, it resembles MQAM locally, i.e., over the transmission of a few symbols. In fact, the QQAM detector behavior is roughly equivalent to an MQAM constellation under a mixture of of SNRs.

The advantage of QQAM is that it removes the effects of channel fading from both the amplitude and phase. Differentially coded modulations such as DQAM still leave the amplitude bits vulnerable to fading, a problem that must be dealt with by equalization techniques such as AGC.[3] As an example, we have examined the relative performances of 32QQAM and 32DQAM. We found that at low E_b/N_0, where the effects of additive noise dominate the fading, DQAM performs better than QQAM, but, at high E_b/N_0, DQAM is fading-limited and falls behind. For the case studied (0.0018 normalized maximum Doppler frequency) there was a crossover point at about 12 dB. Past this point, 32DQAM saturates, and, as a result, QQAM dramatically (by several orders of magnitude) outperforms DQAM.

The intention of this paper was to introduce a modulation technique which, as indicated by simulation, has distinct advantages in fading environments. Many questions remain. A full analytical characterization of QQAM performance would be interesting, but appears difficult. We have not broached the behavior of non-annular geometries or the optimization of performance relative to geometric parameters. A mathematical treatment of the distribution of the magnitudes of the transmitted values could also provide some insights.

[3] An exception, Star DQAM, applies differential coding to the amplitude, but the method is limited to a single amplitude bit.

5 REFERENCES

1. F. Adachi and M. Sawahashi. 1992. "Performance Analysis of Various 16 Level Modulation Schemes Under Rayleigh Fading," *Electronics Letters*, vol. 28, pp. 1579–1581 (Aug).
2. Y. C. Chow, A. R. Nix, and J. P. McGeehan. 1992. "Analysis of 16-APSK Modulation in AWGN and Rayleigh Fading Channel," *Electronics Letters*, vol. 28, pp. 1608–1610(Aug).
3. P. Fines and A. H. Aghvami. 1992. "A Comparison Study of Low Bit Rate DE-QPSK, TCM 8-ary PSK, and 16-ary DE-QAM Fully Digital Demodulators Operating Over a Land Mobile Satellite Link," *IEE Proceedings-I*, vol. 139, pp. 319–328(Jun).
4. W. C. Jakes. 1994. *Microwave Mobile Communications*, IEEE Press, Piscataway, NJ.
5. J. G. Proakis. 1995. *Digital Communications*, McGraw Hill, New York, NY.
6. N. A. B. Svensson. 1995. "On Differentially Encoded Star 16QAM with Differential Detection and Diversity," *IEEE VT*, vol. 44, pp. 586–593(Aug).
7. W. T. Webb, L. Hanzo, and R. Steele. 1991. "Bandwidth efficient QAM schemes for Rayleigh fading channels," *IEE Proceedings-I*, vol. 138, pp. 169–175(Jun).

APPENDIX: Constellations and the Computation of BEP

A complete theoretical treatment of the bit error probability (BEP) for QQAM is well beyond the scope of this paper/study. However, it is enlightening to at least derive some general formulae and outline the relationship of QQAM to the underlying MQAM Constellation. When noise is present, the demodulation takes the form (cf., equation (13)),

$$\frac{q_1 + n_1}{q_2 + n_2}, \tag{A.1}$$

where q_1 and q_2 represent $q(t)$ or its conjugate at two successive times, and n_1 and n_2, the corresponding noise. Assuming large SNR, we may make the approximation,

$$\begin{aligned} \frac{q_1 + n_1}{q_2 + n_2} &= \frac{q_1/q_2 + n_1/q_2}{1 + n_2/q_2} \\ &= \frac{q_1}{q_2} + \frac{n_1}{q_2} - \frac{q_1}{q_2}\frac{n_2}{q_2} + O\left(\left(\frac{n}{q}\right)^2\right) \\ &\approx \frac{q_1}{q_2}\left(1 + \frac{n_1}{q_1} - \frac{n_2}{q_2}\right). \end{aligned} \tag{A.2}$$

Clearly, the factor q_1/q_2 has no effect on the signal-to-noise ratio. In fact, letting σ^2 be the noise variance, and a and b be the magnitudes of q_1 and q_2, respectively, (A.2) implies

$$\begin{aligned} \gamma(a, b) &\triangleq effective\ SNR/symbol \\ &= \frac{1}{(\sigma^2/a^2) + (\sigma^2/b^2)} \\ &= \frac{1}{\sigma^2}\frac{a^2 b^2}{a^2 + b^2} \\ &= \frac{E_s}{\sigma^2}\mu, \end{aligned} \tag{A.3}$$

where

$$E_s \triangleq E(a^2) = E(b^2), \tag{A.4}$$

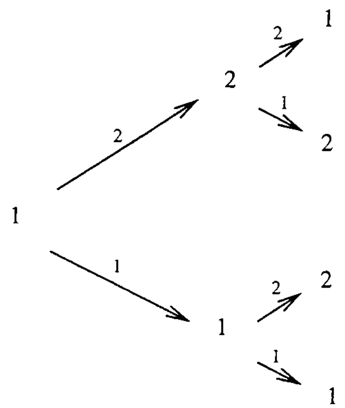
and

$$\mu(a, b) \triangleq \frac{1}{E_s}\frac{a^2 b^2}{a^2 + b^2}. \tag{A.5}$$

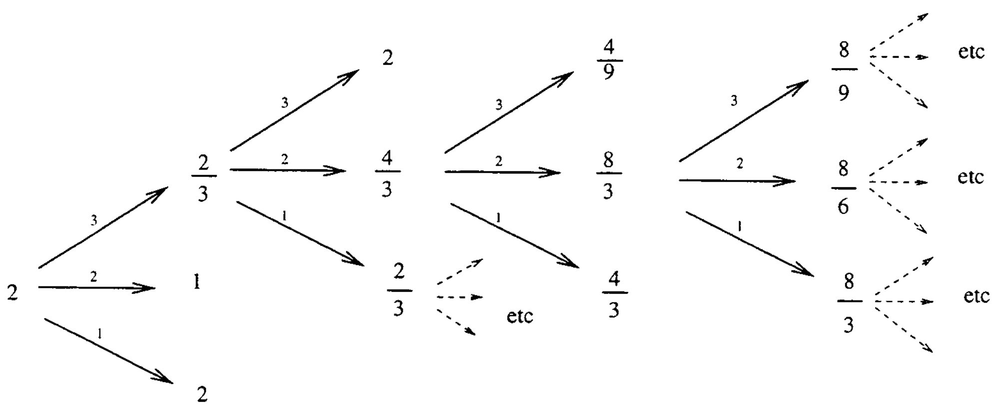
Note that μ is independent of the scaling of a and b; i.e., of any scaling of the signal. The bit error probability, P_B, can be computed by first conditioning on a and b. That is,

$$P_{qqam} = \sum_{a,b} P_B(a, b)\ P(a, b). \tag{A.6}$$

One may consider approximating $P_B(a,b)$ by (a) computing the probability of error under the effective SNR which is $\mu(a,b)(E_b/N_0)$ and (b) by taking into the the probability of an erroneous decision with respect to η of equation (13). However, summation over a and b is much more formidable than one would expect. With very few exceptions (e.g., N = 2), the values of $q(t)$ generated via equation (10) form an infinite Markov chain. More precisely, suppose that N = 2, $A_2 = 2$, and $P_{max} = 4$. Then, the possible transitions from q(t-1) to q(t) are that 1 may go either to 1 or 2, and 2 may go to 2 or 1. This is illustrated in figure 5a. However, for example, if N = 3, then the A_i are 1, 2, 3, and $\sqrt{P_{max}} = 3$, a potential sequence of transitions starting with $q = 2.0$ is 2, 2/3, 4/3, 8/3, 8/9, 16/9, 16/27, 32/27, ... corresponding to transmitted symbols 3, 2, 2, 3, 2, 3, 2, ... (cf., figure 5b). Ultimately, $q(t)$ takes on an infinite number of values.



(a)



(b)

Figure 5. Illustration of propagation and, hence, of the distribution of the values $q(t)$. The small numbers are the symbol magnitudes responsible for the transition. The nodes are the values of q(t). In figure 5(a), $N = 2$ and MQAM amplitudes, $A_1 = 1$ and $A_2 = 2$. In figure 5(b), $N = 3$ and MQAM amplitudes, $A_1 = 1$, $A_2 = 2$, and $A_3 = 3$.

A consequence of this is that the actual received QQAM constellation differs considerably from the underlying MQAM configuration For example, figure 6 is the received constellation (i.e., the eye diagram) for 32QQAM with no noise and no channel fading over 5000 symbols. The constellation fills-in with respect to amplitude (i.e., the spokes). In practice, this poses no difficulty for the detector which really relies on a ratio of amplitudes which, int turn, have the distribution of the underlying MQAM constellation. In fact we see from equations (A.2) to (A.6) that, locally, the detector behaves very similarly to MQAM under a mixture of SNRs. On the other hand, analytically computing its performance is a nontrivial problem.

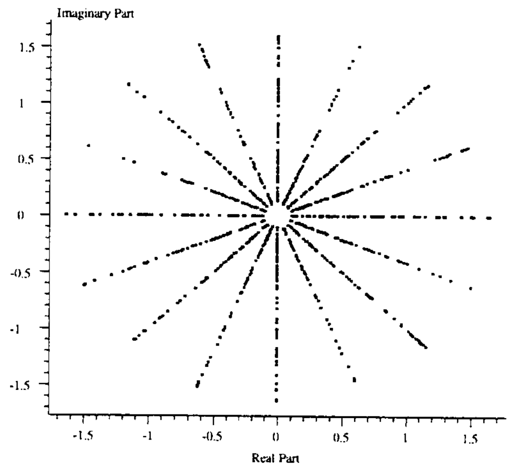


Figure 6. Constellation for 32QQAM (no noise, no fading, 5000 symbols plotted).

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	October 1996	Final

4. TITLE AND SUBTITLE

QUOTIENT QUADRATURE AMPLITUDE MODULATION (QQAM) FOR FADING CHANNELS

5. FUNDING NUMBERS

PE: 0602232N
AN: DN306600
WU: 850-CH73

6. AUTHOR(S)

M. J. Shensa

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

Naval Command, Control and Ocean Surveillance Center (NCCOSC)
RDT&E Division
San Diego, CA 92152–5001

8. PERFORMING ORGANIZATION REPORT NUMBER

NRaD TR 1729

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

Office of Naval Research
800 North Quincy Street
Arlington, VA 22217

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT

Approved for public release; distribution is unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

Multiplicative Rayleigh fading is a frequent problem in wireless communications. If the channel is relatively benign and fading is not severe, one may obtain higher bit rates for an equivalent bandwidth by using M-ary Quadrature Amplitude Modulation (MQAM). A variation, used to combat channel fading while still retaining MQAM, is differential MQAM (i.e., DQAM). The term differential refers to the phase that is coded exactly as in DPSK; however, the amplitude is still subject to distortion by the fading channel. In this paper, we propose a technique called *quotient coding*, which is designed to remove channel effects from the symbol amplitude as well as its phase. In particular, we shall apply it to MQAM resulting in a modulation that we term Quotient Quadrature Amplitude Modulation (QQAM). In contrast to DQAM, QQAM is just as effective at suppressing the effects of channel fading with respect to the entire symbol as DPSK is for the phase alone. Furthermore, it can be applied to arbitrary MQAM configurations.

14. SUBJECT TERMS

Mission area: Command, Control, and Communications
wireless communications
multiplicative Rayleigh fading
quotient coding
differential coding

15. NUMBER OF PAGES

21

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	SAME AS REPORT